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2019

22nd Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 9548

Problem Chosen: B

The Bottle Outbreak

Summary

A bottled water ban prohibits regional retailing of PET bottled water under 500 milliliters in volume. Since its proposal, this act has been the subject of much controversy. While some state it could help by reducing pollution and benefiting consumers, others point out that it might backfire as it disrupts the market. In order to have a full view of the benefits and potential dangers of the ban, multiple factors must be taken into consideration.

The ban has a wide range of impacts, but what it affects directly is the beverage market. The dramatic shift in the market is the fundamental cause for changes at environmental, social, governmental and individual levels.

To predict and analyze the transformations in the market, we have built our major model. This model is based on the Competitive Lotka-Volterra equations, used in biology to simulate species competing for limited resources. Given the similarities between an ecosystem and a market, we modify these equations to simulate competition in the beverage market. We assume the simplest situations at first, considering only two competitors. We then put lower bounds for sales representatively, as specific kinds of beverages usually has their use in life and cannot be entirely "wiped out" over a short period of time. To make the equations more appropriate in a market, we changed them so that they no longer increase exponentially, but in a more linear fashion. Concerning the diversity in a market, we modified our model to make it hold three competitors, and we predicted a possible model for five competitors.

Market impacts lead to further changes. We constructed four branches dependent on our major model. These include environmental, social, governmental and individual changes, and they vary within different contexts. We consider both positive and negative effects. In the context of Concord and San Fransisco, we find that in larger regions, such as a town or a city, energy consumption is larger than that before the ban, though there are minor differences, such as different extra sugar intake after the ban and different taxes applied to the businesses after the ban.

In an airport, however, situations are quite different. The tap water and glass bottled water take up about 70% of the water products market at the airport. The process of competition also takes place more gradually.

In conclusion, we have found that besides the extra costs of installing tap water facilities and the fewer income from taxes, the ban benefits the government and people in many aspects and proved by our model to be a doable ban.

Keywords: Competitive Lotka-Volterra equations

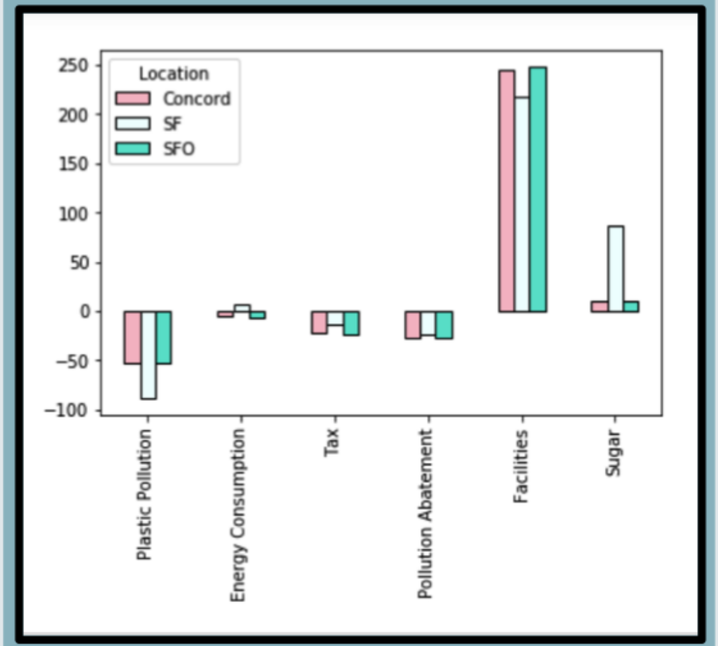
Contents

Newspaper Article

Would The Bottled Water Ban Backfire?

Recently, researchers in Team 9548 has found that the bottled water ban might not work as well as expected.

The ban was a controversial act since its declaration. Now, researchers have used mathematical means to analyze its effects in different situations. Their model, based on the “Lotka-Volterra equations”, predicts the full impacts of the ban in Concord, San Francisco, and SFO airport.



The researchers found that after the ban was implemented, the average sugar intake was higher. Higher risks of diseases caused by extra sugar intake may affect people’s health, according to the researchers.

Good news for businesses--- they will pay fewer taxes to the government since PET bottled water is banned.

For environmentalists, the new ban will, cut the money spent on pollution abatement and release the pressure of severe plastic pollution. In other words, fewer taxes will be paid by individuals and people will be able to enjoy a better environment.

Still, whether the benefits will eventually outweigh the disadvantages or not is yet to be proved by time.

But due to the protection of the government on these necessary data, the researchers aren’t able to get a accurate prediction of the ban. They have to make proper assumptions according to reality.

1 Introduction

1.1 Background

On January 1, 2013, the ban on the use of single-serving PET water bottles came into effect in Concord, making it the first village in the US to do so.

On average, an American spends about 100 dollars on bottled water per year. Though some people argue that bottled water is no better than tap water, it still constitutes 19 percent of the American beverage market, and 67.3 percent of bottled water sold in the US is in single-serve plastic bottles. Technically, PET water bottles are recyclable, but in fact as much as 70 percent of them were not recycled. In the US, thousands of tons of plastic water bottles go to landfills. The government faces the great task of dealing with this waste, adding to its budget and polluting the environment.

Bottled water may not be worth all this trouble. For some brands, their source of water are similar to that of tap water, and they might be close in quality. If people buy bottled water only to feel more comfortable, it seems reasonable to ban it. Concord's bottled water ban has gained quite a bit of attention, and people have praised Concord for its efforts in leading America to combat pollution and minimize waste.

The ban of bottled water, however, is a controversial act. Citizens that argue against the ban says it brings inconvenience. They would have to bring their own bottles with them, and they might still have trouble finding fountains for a refill. Tap water isn't the only alternative, though. Many people choose to buy sugar sweetened drinks or sparkling water instead, as they are unaffected by the ban. Though one or two extra bottles doesn't mean much, drinking soft drinks instead of water on a frequent basis can have a serious impact on an individual's health. If the ban is issued citywide, it might risk producing a regional health issue. Water sold in paper, glass or aluminum containers have also emerged, but they are more expensive compared to PET bottles. Still others criticize the impact of the ban on small businesses, concerning the loss of jobs.

San Francisco and San Francisco Airport have also banned the sale and use of PET water bottles. In comparison with Concord, San Francisco is a city while its airport is a smaller but much more centralized and busy area. For them, situations would vary a little from what they were in the case of Concord.

In Concord, the government has set up drinking fountains for people to refill their bottles. Since their being set up, these fountains have supplied Concord with about a thousand gallons of water, equivalent to 4000 small plastic bottles. In general, Concord's people have, willingly or not, learned to live with the PET bottle ban. However, there is no knowing for sure if the ban is beneficial or harmful when all aspects are taken into consideration, and to what extent.

1.2 Problem Restatement

What the ban directly impacts is the drink market. When PET bottled water exits the beverage market over a short range of time, the previous equilibrium is seriously disrupted. Competitors in the same market, such as soft drinks, would experience a sudden surge in supply and demand. In the end they would reach a new equilibrium, filling the market demand that bottled water previously supplied. The change in the market is the essential reason for further influences in various aspects. We know that it's competitors' sales would increase, but what needs to be further determined is the exact rates by which these sales grow.

We aim to predict the change in a previously balanced market after a major competitor exits it and determine its final state of equilibrium. Based on the change in the market, it will go on to calculate future impacts from environmental, social, governmental and individual aspects. These impacts will depend on the situation in the market.

In different environments, the PET bottle ban has different impacts on the market. By changing the values of variables, the model can simulate airports, towns and cities and adapt to different cases.

2 Assumptions

2.1 Variable declaration

Symbols	Description
s	The money needed to recycle the pollution of bottles made from a single material
a	A conversion factor to calculate the cost of pollution abatement
β	The quantity of water every tap water facility can provide
$Price$	The installation cost of each tap water facility
ΔC_1	Extra costs for individuals to buy other water products
n	The number of bottles
p	The profit rate of the product
C_0	The original cost of the bottle
C'	The changed cost of using bottles of other materials
l_i	How many grams of sugar the drink has in every liter of water
γ	The tendency of people to drink water with additives like sugar
S	The suggested amount of sugar an individual should take in every day
q	The money used on developing a new product line
ΔC_2	The budget spent by the government in emergency supplies
x_i	The ratio of the amount of i^{th} type of specie as a part of the whole
N_i	The possible maximum ratio of the amount of i^{th} type of specie as a part of the whole
t	The time
r_i	The initial natural increasing rate of i^{th} type of species
σ_i	The effect i^{th} kind of species has on another
L_i	The possible minimum ratio of the amount of i^{th} type of specie as a part of the whole
σ_{ij}	The effect j^{th} kind of specie has on another i^{th} kind of specie

Table 1: Symbols and Description

2.2 Assumptions and Justifications

- Assumption: The total demand for bottled water remains constant in a short period of time.
  Justification: Water is a daily necessity for people, so we can assume that each person drinks a fixed amount of water every day. Therefore, the total amount of water consumed in the market can be assumed to be constant. Also, as a necessity, water is a perfectly inelastic good. So the consumer are less sensitive or responsive to a change in the price of water. Any increase in the price results in no decrease in the quantity demanded.

- Assumption: There is a delay between the ban's declaration and execution.
  Justification: The ban wont work the moment it is published. In reality, once the government passes a new policy, it only comes to work after a period of time for the market or people to accommodate to the policy. In our model, the market will have to accommodate to the ban and it takes some time for the market to return to equilibrium. Our model illustrates the process of this by graphing.

- Assumption: The growth of goods in the market is linear instead of exponential.
  Justification: In the Competitive Lotka-Volterra equations, the growth of each species is an exponential growth. But in the real market, the growth isnt exponential as sales wont grow that quickly under competition with other goods. Therefore, the growth of bottled water and other competitors are closer to linear. It is a fairly reasonable assumption because goods in a free market experience a linear growth under common circumstances.

- Assumption: Only three kinds of water goods are taken into calculation of the model.
  Justification: Our model is used to simulate the competition between two products. Later, that number is increased to three. The drinks in the market can be classified into three main types for convenience and to avoid a needlessly high amount of calculation: PET bottled water, packaged drinks, and others (consisting mainly of tap water). Our model is a comprehensive model and can be used in many situations. If more types of goods needs to be taken into consideration, the model can be modified by inputting a different combination of data.

3 Competition Model of the Beverage Market

3.1 the Competitive Lotka-Volterra Equations

The Competitive Lotka-Volterra equations are a simple model of the dynamics in an ecosystem. Commonly used in Biology, they show how two species compete for a resource inside an ecosystem.

Beverage companies seek to maximize their profits. That is, they compete with each other for consumers. As the total number of consumers in a market is relatively stable, they can be thought as competing for a "limited resource". This situation is similar to species competing for food and other resources in an ecosystem. Different kinds of drinks can be compared to different species, and their sales in the market are similar to the numbers of the species inside the ecological community. For two species competing for the same resource, their population dynamics follow such a rule:

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 x_1 (1 - \dfrac{x_1}{N_1} - \sigma_1 \dfrac{x_2}{N_2}) \\ \dfrac{dx_2}{dt} = r_2 x_2 (1 - \dfrac{x_2}{N_2} - \sigma_2 \dfrac{x_1}{N_1}) \end{cases} \tag{1}$$

In the equation shown above, x refers to the total number of a species at present. N is the carrying capacity of a species. σ represents the effect one species has on another. For example, a unit of Species 2 (relative to N_2) uses σ_1 times the resources necessary for a unit of Species 1 (relative to N_1). Likewise, σ_2 stands for the effect Species 1 has on the population of Species 2. r is the inherent per-capita growth rate. This equation demonstrates how the two species compete for a resource, and how that competition influences their populations.

3.2 Modeling the Market With Two Competitors

In the original Volterra model, one of the two competitors usually gains dominance over the ecosystem while the other is completely wiped out.

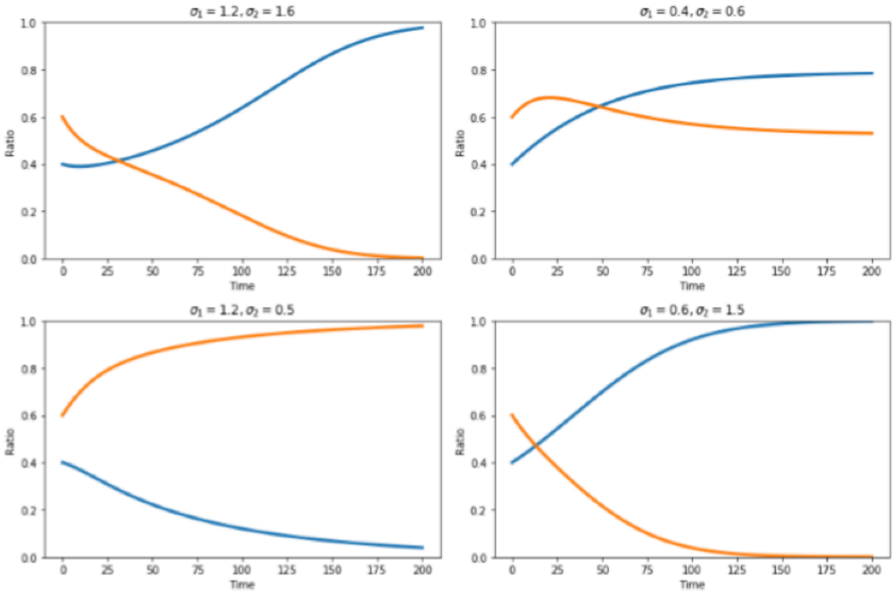


Figure 1: The Situation in the Market Changes with σ Values

As illustrated in Figure 1, the model has four points where it could be stable: $P_1(\bar{N_1}, 0)$, $P_2(0, \bar{N_2})$, $P_3(\frac{N_1(1-\sigma_1)}{1-\sigma_1\sigma_2}, \frac{N_2(1-\sigma_2)}{1-\sigma_1\sigma_2})$, $P_4(0, 0)$. We can see that P_1 and P_2 are situations where all species but one are wiped out. For the species to co-exist, both σ_1 and σ_2 have to be smaller than 1.

In the actual market, a given kind of beverage cannot go "out of business" in a short period of time even if $\sigma > 1$. A beverage may slowly lose popularity, but in general all beverages are of use in specific scenarios. Its sales might be low, but it's unlikely to dwindle to zero. Therefore, we assume a lower bound L for each kind of drink to prevent them from disappearing.

Our modified equation is:

$$\begin{cases} \frac{dx_1}{dt} = r_1 x_1 (x_1 - L_1)(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}) \\ \frac{dx_2}{dt} = r_2 x_2 (x_2 - L_2)(1 - \frac{x_2}{N_2} - \sigma_2 \frac{x_1}{N_1}) \end{cases} \tag{2}$$

After we modify the model, our model can reach a reasonable equilibrium, and the sale of each kind of drinks will never be below the minimum amount. The model can better illustrate the real situation of the market.

3.3 Modeling the market with three competitors

The model can be further improved. Since in reality there are more than 2 kinds of drinks, a model that could take more than 2 kinds of drinks into consideration should be constructed. As considering too many types of beverages makes calculations needlessly complex, we consider two major components of the market: bottled water and packaged drinks, such as soft drinks. Other drinks are treated as a whole. We assume that x_1 is the percentage of PET Bottled Water in the market, x_2 the percentage of Packaged Drinks and x_3

that of Other Drinks. We assume that these kinds of drinks compete in a free market with positive initial growth rates. Now, they affect each other and their percentage of the market begins to change according to the equation below:

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 x_1 (x_1 - L_1)(1 - \dfrac{x_1}{N_1} - \sigma_1 \dfrac{x_2}{N_2}) \\ \dfrac{dx_2}{dt} = r_2 x_2 (x_2 - L_2)(1 - \dfrac{x_2}{N_2} - \sigma_2 \dfrac{x_1}{N_1}) \\ x_1(t) + x_2(t) + x_3(t) = 1 \end{cases} \tag{3}$$

Since water is a daily necessity, we assume that the total demand of drinks is constant. "Other drinks" consist mainly of tap water.

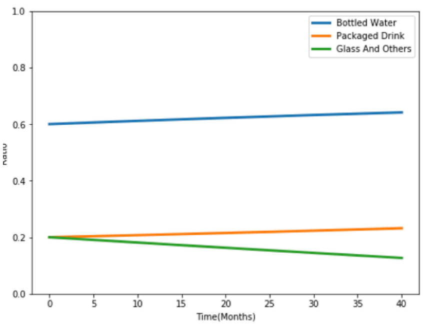


Figure 2: The Market Before the Ban

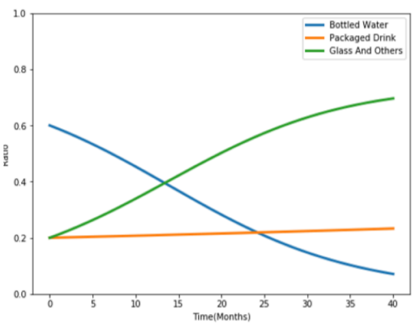


Figure 3: The Market After the Ban

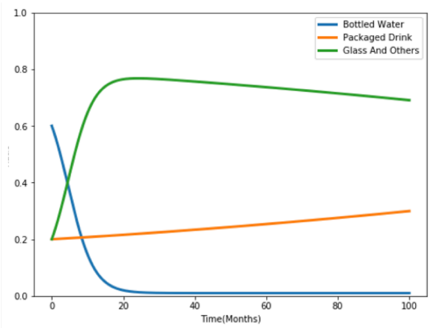


Figure 4: What Happens If the Delay Is Shortened

As shown in Figure 2, before the ban, the percentage of these kinds of drinks stays roughly the same.

A great disruption in the market occurs soon after the ban is declared. Companies, being aware that selling bottled water will no longer be legal, stop their manufacture and sell those left in stock. While the ban is declared but not yet enforced, people can still choose to buy the bottled water in stock. Therefore, sales of bottled water will decline at a considerable speed, but they will not suddenly drop to zero and cause the situation shown in Figure 3.

To let the number decrease smoothly, we assume the initial per-capita growth rate of bottled water to be -0.1. This enables the bottled water to decrease at a reasonable speed.

While the sale of bottled water decreases, other kinds of drinks fill in the gaps, ultimately resulting in a new equilibrium, which is shown in Figure 4. If the ban is declared only shortly before its implementation, this balance can be reached in a shorter time.

3.4 Predicting The Market with Five Competitors

The market is diverse. With three competitors we can get a big idea of what happens after the ban, but it can be hard to derive more detailed analyzation from that model. We have mentioned before that complexity increases exponentially with the number of competitors, making this sector very difficult to calculate rigorously. However, some underlying correlation can be inferred from what has already been done.

The major barrier lies in estimating reasonable coefficients. Though they are "deducted" in our model, in reality they can be obtained by surveys. As tap water is generally available to everybody, we assume that consumers choose it when none of the other drinks please them. That is, tap water fills in the "gaps" in the market.

In the final part of our model, we sort beverages into five major kinds. The association between their sales can be specified as:

$$\begin{cases} \frac{dx_1}{dt} = r_1(x_1(t) - L)(1 - \frac{x_1(t)}{N_1} - \frac{\sigma_{1-2}x_2(t)}{N_2} - \frac{\sigma_{1-3}x_3(t)}{N_3} - \frac{\sigma_{1-4}x_4(t)}{N_4} - \frac{\sigma_{1-5}x_5(t)}{N_5}) \\ \frac{dx_2}{dt} = r_2(x_2(t) - L)(1 - \frac{x_2(t)}{N_2} - \frac{\sigma_{2-1}x_1(t)}{N_1} - \frac{\sigma_{2-3}x_3(t)}{N_3} - \frac{\sigma_{2-4}x_4(t)}{N_4} - \frac{\sigma_{2-5}x_5(t)}{N_5}) \\ \frac{dx_3}{dt} = r_3(x_3(t) - L)(1 - \frac{x_3(t)}{N_3} - \frac{\sigma_{3-1}x_1(t)}{N_1} - \frac{\sigma_{3-2}x_2(t)}{N_2} - \frac{\sigma_{3-4}x_4(t)}{N_4} - \frac{\sigma_{3-5}x_5(t)}{N_5}) \\ \frac{dx_4}{dt} = r_4(x_4(t) - L)(1 - \frac{x_4(t)}{N_4} - \frac{\sigma_{4-1}x_1(t)}{N_1} - \frac{\sigma_{4-2}x_2(t)}{N_2} - \frac{\sigma_{4-3}x_3(t)}{N_3} - \frac{\sigma_{4-5}x_5(t)}{N_5}) \\ x_1(t) + x_2(t) + x_3(t) + x_4(t) + x_5(t) = 1 \end{cases} \tag{4}$$

The x_1, x_2, x_3, x_4, and x_5 shown above represents the percentages of different kind of drinks sold. They are functions of time t.

- x_1: The percentage of single-serving PET bottles of water with a volume of less than 1 L sold.
- x_2: The percentage of single-serving PET bottles of water with a volume of equal to or greater than 1 L sold.
- x_3: The percentage of beverages (drinks that are not water) sold.
- x_4: The percentage of reusable glass bottles of water sold.
- x_5: The percentage of liters of tap water sold.

Among them, x_1 is what would be affected in the bottled water ban.

σ values are impediment coefficients. They represent how the products compete with each other in a market. The coefficients are only meaningful when they are related to a pair of variables.

r is the rate of increase for each kind of drink, which is a constant. As the total demand in the beverage market is limited, each variable mentioned above are related to a N. N is the maximum possible percentage for one kind of beverage.

Each kind of drinks competes with each other for customers in order to maximize profit. The percentages of these kinds of drinks sold keeps increasing and comes near to a constant. This constant is the theoretical maximum and can never be reached.

As mentioned, x_1, x_2, x_3, x_4, and x_5 are functions of time. What we do now is to make a calculation every small period of time so that these dots nearly make up a curve. In every calculation, new data is generated according to the most recent numbers. That is, every new "dot" is dependent on its predecessor. When the value of t increases to an extent, we will come to a situation in which the dependent variables of the functions are very close to the theoretical maximum. They increase so slowly that they can nearly be considered stable. The market has reached an equilibrium. This can be considered as the "constant" situation before the ban takes place. We dub it the "first stable period".

Now, the ban takes place. As a result of the ban, x_1 no longer increases. That is to say, the r for x_1 becomes a negative value. As the number of single-serving water bottles less than 1L decreases, other kinds of drinks will gain consumers, and their sales will increase. In this part, we assume that x_1 will decrease by a smooth curve instead of directly dropping to zero.

We call this part the turning period. In this part, x_1 will decrease smoothly but sharply. The stable period before the ban exists for a long time. In comparison, the turning period is much shorter.

At the end of the turning period, single-serving water bottles less than 1L finally disappears, and x_1 reaches zero. Now, it no longer affects the sale of other products because its value stays at zero. We no longer need to take x_1 into consideration. Thus, we only need to predict the changes among other amounts.

$$\begin{cases} \frac{dx_2}{dt} = r_2(x_2(t) - L)(1 - \frac{x_2(t)}{N_2} - \frac{\sigma_{2-1}x_1(t)}{N_1} - \frac{\sigma_{2-3}x_3(t)}{N_3} - \frac{\sigma_{2-4}x_4(t)}{N_4} - \frac{\sigma_{2-5}x_5(t)}{N_5}) \\ \frac{dx_3}{dt} = r_3(x_3(t) - L)(1 - \frac{x_3(t)}{N_3} - \frac{\sigma_{3-1}x_1(t)}{N_1} - \frac{\sigma_{3-2}x_2(t)}{N_2} - \frac{\sigma_{3-4}x_4(t)}{N_4} - \frac{\sigma_{3-5}x_5(t)}{N_5}) \\ \frac{dx_4}{dt} = r_4(x_4(t) - L)(1 - \frac{x_4(t)}{N_4} - \frac{\sigma_{4-1}x_1(t)}{N_1} - \frac{\sigma_{4-2}x_2(t)}{N_2} - \frac{\sigma_{4-3}x_3(t)}{N_3} - \frac{\sigma_{4-5}x_5(t)}{N_5}) \\ x_2(t) + x_3(t) + x_4(t) + x_5(t) = 1 \end{cases} \tag{5}$$

x_1 is eliminated from the equations. They eventually lead us to another theoretical maximum. We call this period of time the second stable period. In this part, the sales of other drinks increase in a pattern similar to what happened in the first stable period. When their rates of increase slow down to a negligible point, we assume that they have reached an equilibrium and record the data.

Then, we compare the data at the end of the first stable period with that of the second stable period. These two groups of data are the stable situations before and after the ban. We analyze them and find out the bans advantages and disadvantages.

In the program, we calculate the whole procedure from the beginning to the final stable situation, and the result is shown in Figure 5:

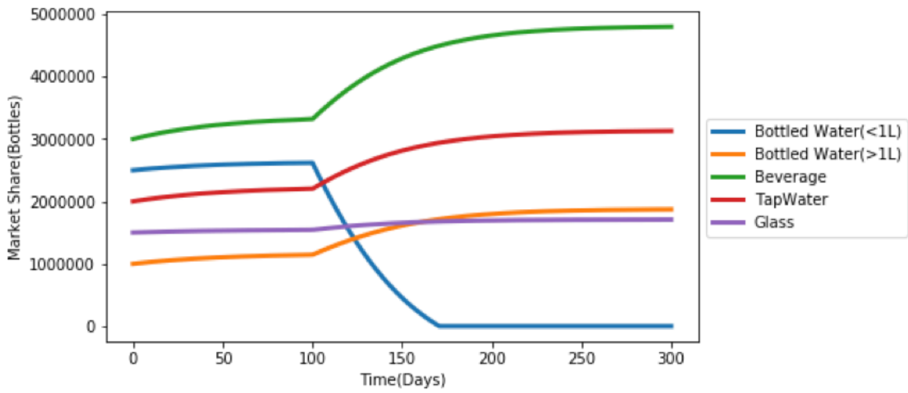


Figure 5: Changes in the sales of different beverages.

4 The Four Impacts of the Bottled Water Ban

4.1 Environment

The first impact of the ban is on the environment. This includes both plastic pollution and energy consumption in manufacturing the bottles.

4.1.1 Plastic Pollution

Not all PET bottles thrown away are recycled. Some bottles are thrown into the wrong rubbish bins and will be just buried in landfills. Besides, after drinking all the water in PET bottles, not everyone will throw the bottle into the rubbish bins. Some bottles are left in places like the forest or thrown into the sea. These two factors contribute to the plastic pollution caused by PET bottles. After the ban is applied, the plastic pollution caused by PET bottles will no longer exist.

The mass of waste plastic M reduced per month can be calculated by using the equation:

$$M = \sum_{i=0}^{n} (1 - R) P_i M_i \tag{6}$$

The variable R represents the recycled portion of the bottles. The variable p_i means the change in the number of liters of the $i^{th}(i1)$ types of drinks sold after the ban is applied per month. The variable $m_i(i1)$ represents the mass of plastic required to contain one liter of the $i^{th}(i1)$ types of drinks. We specify p_0 to represent the change in the number of liters of the PET bottled water sold after the ban is applied per month. And we specify m_0 to represent the mass of PET material required to contain one liter of water.

We can find the actual quantity of the variable R_i, p_i, m_i on the Internet or by interviewing related government officials.

4.1.2 Reduction in Energy Consumption

The factories that produce PET bottled water need to transport the water from rivers and lakes, produce the bottles and use trucks to transport raw materials to the factories and products to the dealers. The manufacturing process consumes a lot of electricity and trucks burn a considerable amount of fuel. Therefore, the production and the transportation of PET bottled water require a lot of energy and intensify global warming.

On the other hand, the recycle of the PET bottles can transform PET bottles into raw materials like chemical fiber. This can save some energy and compensate for part of the energy consumption.

After the ban is applied, this energy can be saved.

We can estimate the reduction in energy consumption by calculating the carbon footprint (the mass of the emitted carbon dioxide to produce the required amount of the energy) caused by the PET bottled water per month M:

$$M = \frac{p_i(x_i - RE_i)}{k} \tag{7}$$

The variable p_i means the change in the number of liters of the $i^(th)(i1)$ types of drinks sold after the ban is applied per month. The constant x_i represents the energy required to produce one liter of the $i^(th)(i1)$ types of drinks. The variable R represents the recycling rate of plastic bottles. The constant E_i means the energy saved by recycling the plastic which is used to contain a liter of $i^(th)(i1)$ types of drinks. The constant k is a conversion factor that is used to calculate the carbon footprint. We specify p_0 to represent the change in the number of liters of the PET bottled water sold after the ban is applied per month. We specify x_0 to represent the energy required to produce one liter of PET bottled water. We specify E_0 to represent the energy saved by recycling PET material which is used to contain a liter of water.

We can find the quantity of x_i, E_i and k in scientific reports.

4.2 Social Impact

The second impact of the ban is social and consists of change in tax revenues and employment.

4.2.1 Tax

Dealers of the bottled drinks have to pay some taxes to the government. Each kind of drink has a different tax standard. With the ban on selling water in PET bottles, the tax on the water in PET bottles no longer exists while the sales of other kinds of drinks may increase and the dealers need to pay more taxes for selling them. To the government, the impact will be positive if the revenue of the sales tax increases and will be negative if the tax revenue decreases.

The change of the tax revenue R per month after the ban is applied is:

$$R = \sum_{i=1}^{n} p_i q_i - p_0 m \tag{8}$$

We suppose there are n kinds of drinks in PET bottles other than water. The constant p_i represents the tax that needs to be paid per liter of the $i^{(th)}$ kind of drinks sold. The variable q_i represents the change in the number of liters of the i^{th} kind of drink sold per month. The variable p_0 means the number of liters of the water in PET bottles sold before the ban is applied per month. The constant m indicates the tax that needs to be paid per liter of the water in PET bottles sold before the ban is applied.

We can find the quantity of p_i, q_i and m in the website of local taxation administration.

4.2.2 Job Change

With the ban on sales of water in PET bottles, the workers who produce this kind of bottled water are no longer needed in the factory. They would lose their jobs. This is a negative effect.

The number of people who will lose their jobs N after the ban is applied can be estimated as:

$$N = \frac{p_0}{w} \tag{9}$$

The variable p_0 represents the number of liters of the water in PET bottles sold before the ban is applied per month. The variable w means the average number of bottles of water in PET bottles that a worker can produce per month.

We can find the quantity of w by interviewing the person (once) in charge of the production of the PET bottles.

4.3 Government Budget

The third impact of the ban is an increase in government budget on both pollution abatement and setting up tap water facilities.

4.3.1 Pollution Abatement

Since PET materials are banned from bottled water, the government will spend money on the recycle of the increasing amount of other materials. The regulation of different materials differs from each other but is constant. So the total cost will change due to the ban on PET materials and the increase in other types of materials. The budget is an important factor in judging the ban so we need to quantify the budget.

For pollution abatement, we have this equation to explain how the budget will change:

$$W = \sum Dx_i s_i \tag{10}$$

W refers to the budget of the government, i refers to the material of the bottle, S refers to the number of the bottle and m refers to the money needed to control the pollution of bottles made from a single material.

From this equation, we can know that the total budget of the government is related to the amount of each type of bottles and the money required on the disposal of landfill. If the ban is implemented, the government dont need to recycle bottles made of PET but more bottles of other materials. The number of bottles made of other materials increases while the cost of recycling these bottles remains the same. Therefore, we can work out how much the government needs to pay once the ban is implemented and judge whether the impact is positive or not.

4.3.2 Tap Water Facilities

To make up for the ban of PET bottled water, the government also has to install more free tap water facilities. This also adds up to the budget of the government. If people dont want to spend extra money to buy bottled water of other expensive materials, the government has to think of a plan to meet their needs to drink. More tap water facilities will be built. Tap water is a free resource of water for people along the street that enables them to drink water when they want to. The extra cost caused by this act needs to be counted as well to accurately calculate the total budget of the government.

For setting up tap water facilities, this equation can be used to calculate the installation cost:

$$N = \left[\frac{Dx_3}{\beta}\right] Price \tag{11}$$

in which N refers to the budget spent on installation of the facilities, $Price$ refers to the installation cost of each tap water facility, β refers to the quantity of water every tap water facility can provide and $\left[\frac{Dx_3}{\beta}\right]$ refers to the number of newly installed tap water facilities.

The government has to do extra acts to regulate the market to make the market return to stability as soon as possible and to meet peoples need to drink pure water. Therefore, the government surely will implement more tap water facilities across the city. The cost of this act can be calculated by the equation above as the cost of each installation is a constant and equation is a proportional function.

The cost goes up in direct proportion to the number of tap water facilities. From this equation we can work out how much the government needs to pay on the set-up of the facility and evaluate the impact.

4.4 Individual Issues

The fourth impact of the ban is inconvenience brought to individuals using bottled water. The inconvenience includes the change in individual cost, health, and emergency supplies.

4.4.1 Individual Costs

For individual costs, if the injunction is implemented, the number of PET bottled water will decrease and finally disappear from the market. PET material is non-toxic, odorless, hygienic and safe, and can be directly used in food packaging. It has low gas and water vapor permeability, excellent gas barrier, water, oil and odor properties. Rather high and low temperatures have little effect on its properties. Therefore, PET is a preferable material for water bottles. Banning the sale of PET bottled water force the suppliers to use other materials, which increase the cost of manufacturing a single bottle of water. The cost for an individual to drink water in turn increases. We can use this equation to calculate the extra cost:

$$\Delta C_1 = np(C' - C_0) \tag{12}$$

in which ΔC refers to extra costs, n refers to the number of bottles, p refers to the profit rate of the product, C_0 refers to the original cost of the bottle and C' refers to the changed cost of bottles of other materials.

4.4.2 Health

For health, the ban may lead to more additives such as sugar in the water to avoid being banned. This will increase few costs of manufacture and provide customers with relatively lower-price products in comparison with other options of pure water. The customers tend to buy these products as they can enjoy the similar taste of pure water and lower prices. But if they continue to drink this kind of additive water, the added additives will lead to physical problems. For example, obesity rate will increase if people keep drinking water with more sugar. Obesity will cause other severe physical problems such as diabetes mellitus, hypertension, high blood sugar and so on.

For how this extra sugar intake will affect people, we can use this equation to evaluate:

$$Q = max(\sum Dx_i(l_i - S), 0) \tag{13}$$

in which Q means how seriously will people be affected, l_i refers to how many grams of sugar the drink has in every liter of water, S refers to the suggested amount of sugar an individual should take in every day. The higher the magnitude of Q is, the higher rate of diseases will be.

4.4.3 Emergency Supplies

For emergency supplies, the government have warehouse for emergency supplies or need to transport supplies to a stricken area. The government have bought bottled water ahead of time for sake of disasters. If PET bottles are prohibited, the government has to pay more to instore the same amount of water. Although there is no ban on PET bottled water, bottled water under 1L is still needed because sanitation is extremely important in a stricken area. If people there are only provided with large bottles of water, they have to share the same bottle with each other. It might spread diseases over that area and make the situation worse. For this extra cost of buying these supplies per bottle, we can use this equation:

$$\Delta C_1 = np(C' - C_0) \tag{14}$$

In another situation, the government has to customize the supplies rather than buying them from manufacturers. Emergency supplies are exceptions of the ban because stricken areas need these supplies to keep people there alive. Therefore, the government shall contact factories and ask them to produce PET bottled water and sell the product only to the government for emergency use. There is extra cost than buying PET bottled water because the factory has to develop a new product line to produce, which definitely will cost much more. After starting a new product line, the government then can buy PET bottled water from the factory. We can use this equation to calculate this kind of extra cost:

$$\Delta C_2 = q + np(C' - C_0) \tag{15}$$

in which ΔC_2 refers to the budget spent, q refers to the money used on developing a new product line, n refers to the number of bottles, p refers to the profit rate of the product, C_0 refers to the original cost of the bottle and C' refers to the changed cost of bottles of other materials.

5 Model Application

5.1 Our Model for a City: Concord and San Francisco

5.1.1 Environment

People in cities can throw the PET bottles into the rubbish bins and classify them correctly in most cases. There are also sufficient rubbish bins around. Therefore, while calculating the reduction in plastic pollution or reduction in energy consumption, we can choose 0.95 as the value of the recycling rate of the PET bottles R.

5.1.2 Social impact

Concord is a small town with an approximate population of 130,000. The demand of bottled water is lower in small towns. We can estimate that every citizen in Concord consumes about 0.1 PET bottles on average every day. And we assume that the volume of each PET bottle is 500 milliliters. Therefore, we can use 195,000 as the value of the number of liters of the water in PET bottles sold before the ban is applied per month p_0.

San Francisco is a big city. The demand of bottled water is higher in big cities. We can estimate that every citizen in San Francisco consumes about 0.15 PET bottles of water on average every day. And San Francisco has a population of 880,000. We assume that the volume of each PET bottle is 500 milliliters. Therefore, we can use 1,980,000 as the value of the number of liters of the water in PET bottles sold before the ban is applied per month p_0.

5.1.3 Government Budget

For these specific situations we can use this modified equation to calculate the outcome:

$$w = a \sum S_i m_i \tag{16}$$

For Concord, there arent that many plants used to abate pollutions as it is a small city with lower average income than other cities in New Hampshire. If PET bottled water is banned, the government will have to build factories to do pollution regulation. Because there will be extra cost building new plants, here we suppose that a is larger than other two situations so we assume that a equals 1.

For San Francisco, there have already been large number of plants for different types of plastic and garbage. It doesnt need to build more plants as Concord so a in this case should be smaller than in the case of Concord. We assume that a here equals 0.5.

5.1.4 Individual Issues

In cities, the extra cost for people to meet their need of drinking water remains the same and can be calculated by the equation previously stated.

$$\Delta C_1 = np(C' - C_0) \tag{17}$$

Since the cost for factories to produce these products is nearly the same everywhere, the profit rate for all businesses is also similar. Thus p can be assumed to be 0.5.

For individual health, we can modify the formula into this:

$$Q = max(\gamma \sum Dx_i(l_i - S), 0) \tag{18}$$

in which γ refers to the tendency of people to drink water with additives like sugar.

For Concord, the people there may not choose pure water in other kinds of packaging because they may cost more than water that contain some additives. So we shall give γ here a larger value as 1

For San Francisco, people there care more about health problems as they have more money to put into taking care of their health. So their tendency of buying waters with additives is lower than that of people in Concord. The value of in this case can be assumed as 0.1.

In emergency supplies, the cost for warehouse storage is same for both cities because the cost of buying bottled water of other materials is similar for both cities.

5.2 Our Model for an Airport: San Francisco Airport

5.2.1 Environment

People can find the bins for recyclable waste in the San Francisco Airport easily and airports and airports are often equipped with cleaners. We can assume that everyone in an airport throws the PET bottles into the "recycle" bin. Thus, all PET bottles in the airport can be recycled, and plastic pollution does not exist in this situation.

Since all PET bottles are recycled in airports, the recycled portion of the PET bottles R equals 1. So, the variable R is no longer needed in the equation of calculating carbon footprint.

The equation used to calculate the reduction in energy consumption in an airport is:

$$M = (p_0(x - E))/k \tag{19}$$

5.2.2 Social impact

Both the change in the tax revenue and unemployment rates are only affected by the decline in the sale of water in PET bottles, which changes the number of liters of different drinks sold per month.They are independent of the change of the area. Therefore the model of social impact remains unchanged in San Francisco Airport.

5.2.3 Government Budget

For San Francisco Airport, the cost for abatement in the airport does not have a huge change because the airport shares almost the same situation as normal one. So we assume that a here equals 0.1.

5.2.4 Individual Issues

For San Francisco Airport, the businesses there have to pay more rent because they are in the airport. Therefore, the bottled water there costs more than those in the city. The profit rate of bottled water in the airport can be set as 1 to be different from that in Concord or San Francisco.

For San Francisco airport, there is more dessert shop and cafeteria and these stores provide less bottled water. People used to buy bottled water from these shops and if them no longer provides bottled water but water with additives, people will take in more sugar than regular. Therefore, here can be set as 0.5.

In emergency prevention, the cost of warehouse water storage is 0 because the government will make preparation for emergency use for the airport and the airport doesnt have to spend extra money on this.

6 Analysis and Suggestions

Though we have constructed complete models fit for all three scenarios, there are a few equations that we did not analyze in detail for various reasons.

- Possible Job Change

  All factories use machines to produce PET bottled water. A machine can produce over 45,000 bottles of water in an hour and only requires about 8 workers to control it. Therefore, only a few people will lose their jobs and the impact will be negligible. We will not discuss the impact of the job losses in the application of Concord, San Francisco, and San Francisco airport.

- Individual Costs & Emergency Supplies

  It is very difficult for us to find the data of profit rate, the original costs and the changed costs in the limited time. Therefore, we will not discuss these two impacts in the model application of Concord, San Francisco, and San Francisco Airport. However, we are sure that we can calculate the change in individual costs and the cost of government emergency supplies if the data is given to us.

6.1 Concord

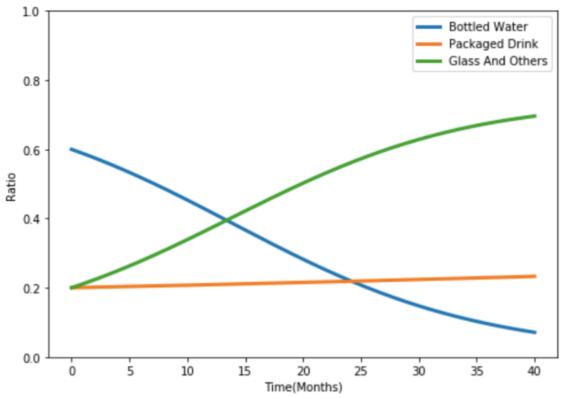


Figure 6: The Market in Concord After the Ban is Declared

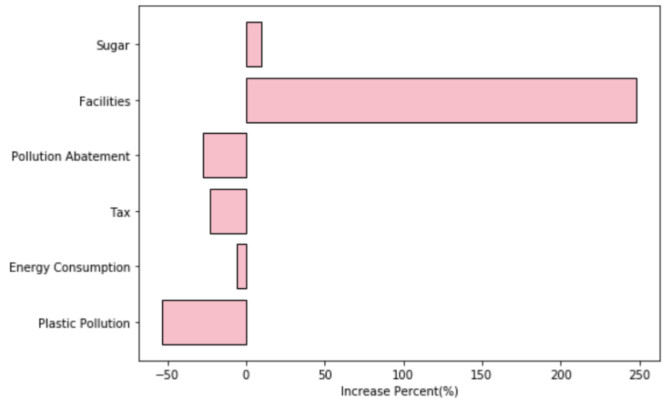


Figure 7: The Benefits and Harms of the Ban on Concord

Since the government didn't public the details, we can only guess the consistence of the market in Concord. We assume that at first, the ratio of the amount of bottled water, packaged water and others in the market is 6:2:2. Meanwhile, the ban only ban part of the bottled water, and let the initial natural increasing rate decrease 0.1. We also assume that the total need in the market is 12000 and 95% of the plastic bottles can be recycled.

According to the model of calculating the impacts we mentioned before, we can now get the number of several important data.

As can be seen from Figure 6 and the data in Table 3 shown below, after the ban acts, the data changes fiercely. Less bottled water leads to less plastic bottle, and decrease the use of plastic. This can help to protect the environment for plastic pollution is a chief part in todays environmental problems. Meanwhile we can use less money to deal with the pollution. Producing fewer plastic bottles also decrease the energy used in producing. In our model, we assume the tax rate, and we find that less tax will be get by the government. This means less money will the citizens use to hand in tax, and less money can the government receive also. Finally, since less bottled water are used, the citizens may buy other drinks that contains sugar instead. This will lead to that more sugar will be obtained and do harm to the health.

As shown in Figure 7, the ban had a fairly good effect in reducing pollution. Correspondingly, the government's cost of pollution abatement is lowered as well, but it is not enough to make up for the decrease in tax revenue. As more people turn to tap water which is tax-free, government tax revenue has decreased. In addition, the government needs to pay

Item Name	Before the Ban	After the Ban
Plastic Pollution (t)	19.8	9.25
Energy Consumption (kJ)	13849.0	130004.9
Tax (k$)	4400.0	3381.3
Pollution Abatement (k$)	158.4	114.7
Facilities (k$)	176.0	612.4
Sugar (g/Day)	4.4	4.8

Table 2: Concord

for the construction of public water fountains. The ban has a minimal effect on health as only a small percent of people have substituted bottled water for sweet, packaged drinks.

To sum it up, the bottled water ban cost the government dearly, but it helps the environment and reduces energy consumption.

For legislators, we recommend them to increase taxes on both beverages and tap water to pay the cost of installing facilities. "Taxing" tap water can be done by charging a minimal fee to use public water fountain facilities. This is a reasonable thing to do as the people using water fountains benefit from their convenience. Beverages need to be taxed as well to increase their price and prevent citizens from choosing unhealthy drinks against tap water.

6.2 San Francisco

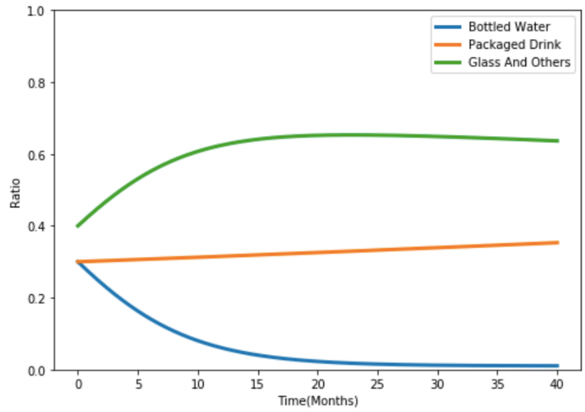


Figure 8: The Market in San Francisco After the Ban is Declared

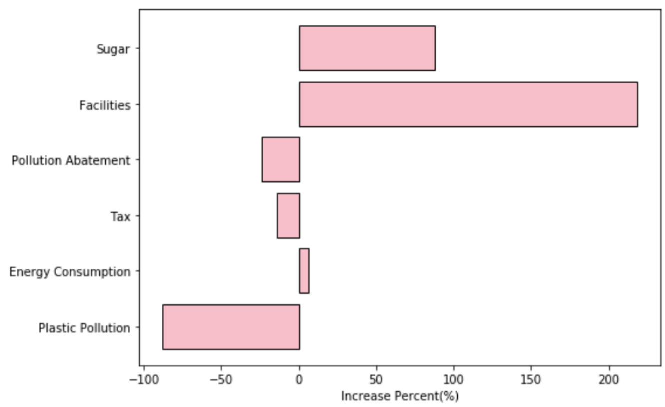


Figure 9: The Benefits and Harms of the Ban on San Francisco

The people in San Francisco have a higher demand for PET bottled water before the ban is applied. So, as shown in Figure 8, the result in San Francisco shows that the sale of the drinks in other packages like glass increases relatively slower than that in Concord and the sale of water in PET bottles also decreases relatively slower. People in San Francisco can throw the PET bottles into the rubbish bins and classify them correctly in most cases. There are also sufficient rubbish bins in San Francisco. Therefore, while calculating the reduction in plastic pollution or the reduction in energy consumption, we can choose 0.95 as the value of the recycling rate of the PET bottles R.

According to Figure 9, as there are already plenty of tap water facilities in San Francisco, the increase rate of the cost is relatively low.

Item Name	Before the Ban	After the Ban
Plastic Pollution (t)	19.8	2.4
Energy Consumption (kJ)	13849.0	14691.5
Tax (k$)	4400.0	3776.5
Pollution Abatement (k$)	158.4	119.984
Facilities (k$)	176.0	560.16
Sugar (g/Day)	4.4	8.3

Table 3: San Francisco

Many people in San Francisco would like to choose drinks with additives like sugar to take the place of PET bottled water and more drinks with additives will be produced. Therefore, according to Table 3, they take in more sugar and the energy consumption increase. But the increase in the sale of drinks with additives also contributes to a relatively lower decrease rate in the tax revenue.

Another notable thing is that the plastic pollution rate drops greatly in San Francisco. It decreases by about 80%. This is a very positive effect.

We can see that despite considerably cutting pollution, the ban has caused San Francisco's sugar consumption to increase greatly, with packaged drinks accounting for more than a third of the market. Facilities also cost much, with fees more than tripling. Due to people's choosing packaged drinks, energy consumption has even risen a little. On the bright side, pollution and the cost of regulating it have dropped, and taxes aren't greatly affected.

In general, a plastic bottle ban might not be such a good idea for San Francisco, though it undeniably has its benefits.

We recommend policymakers to encourage people to choose tap water over packaged drinks. This can reduce sugar intake and further reduce pollution. This can be done by warning the public against harmful effects of sugar, taxing sweetened beverage manufacturers, and refurnishing public water fountains to make them look attractive and safe to drink from.

6.3 San Francisco Airport

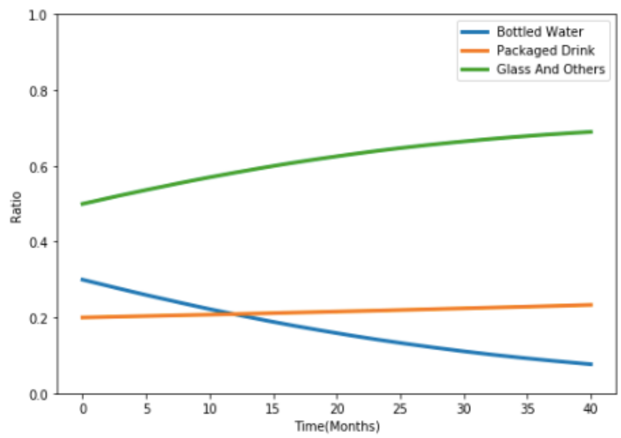


Figure 10: The Market in San Francisco Airport After the Ban is Declared

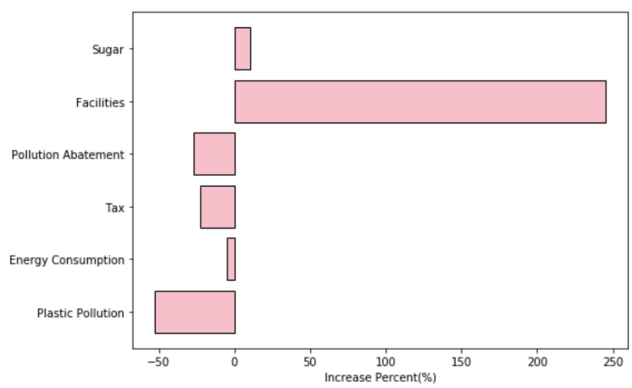


Figure 11: The Benefits and Harms of the Ban on San Francisco Airport

Item Name	Before the Ban	After the Ban
Plastic Pollution (t)	3.96	1.88
Energy Consumption (kJ)	13505.8	12860.5
Tax (k$)	4400.0	3395.5
Pollution Abatement (k$)	158.4	115.28
Facilities (k$)	176.0	607.16
Sugar (g/Day)	4.4	4.84

Table 4: San Francisco Airport

In the case of San Francisco Airport, situations are different from those in the town or city because its an airtight area with an extremely large flow of people. All the trades happened at the airport are small-scale trades between individuals and businesses. As there are plenty of tap water facilities at such a relatively small area, peoples need to drink water can be filled by tap water rather than water with additives. As shown in Figure 10, according to the result of our model, the sale of bottled water will gradually drop and reach 0. The need for packaged drinks doesnt increase too much due to the existence of tap water facilities. The use of glass bottled water, tap water, and other water products rise obviously by nearly 250% and the ratio of this type of water products is around 0.7. According to Figure 11 and Table 4, for the 6 dependent impacts on different aspects, the recycling rate of plastic at the airport is 1 because all the garbage created can be collected and recycled. Also, the energy consumption at the airport is lower than before the ban. It is easier to collect and transport wasted plastic to recycle plants. Due to the larger amounts of tap water facilities installed, the cost for these devices is relatively higher than in the other two situations and plastic pollution drops by nearly 50%. Besides, taxes for businesses and pollution abatement cost for the government is lower than that before the ban. For the government, it only needs to pay for the extra tap water facilities and relatively fewer taxes then gain improvement in other different aspects.

The major disadvantage of the bottled water ban on San Fransisco Airport is still in the costly construction of facilities, considering the fact that public water fountains are the only resources of obtaining drinking water in an airport. Fewer taxes are collected, but the reduction percentage is acceptable. Sugar and energy consumption doesn't change much. Pollution and costs of pollution abatement are reduced significantly.

As sugar consumption does not greatly increase after the ban, it is possible that most people prefer tap water to unhealthy drinks. Therefore airport administrators can consider placing water fountains wisely. They can arrange their locations so that fewer fountains are needed. This can cut the cost for the facilities.

7 Strengths and Weaknesses

7.1 Strengths

Our main model is based on the traditional Volterra equations. This provides us with a steady theoretical basis. The market competition is very similar to the competition in species in many ways. With relative accuracy, we can use this model to estimate the quantity of the different drinks that will be sold after the ban is applied.

We do not only consider the environmental influence. Our model evaluates the impact of the ban on all aspects. This is very important because the environment is not the only thing that people need to consider in real life. People also care about things like government budget and unemployment. Our model quantifies the impact of these aspects.

Therefore, our model has potential value in reality. It could help policymakers to evaluate the impact of a new unapplied policy and decide whether to apply it or not. We can say that our model is designed with a purpose.

7.2 Weaknesses

Our model still has some disadvantages.

Since we do not have enough authentic data, our model might have some deviation. The data is important when we need to determine the coefficients in our model. Without enough data, those factors might not be the most fitting ones.

The market competition is not completely the same as the competition in species. The result of the competition in the products is not only determined by the competition itself but is also influenced by factors such as advertisment.

In general, our model is nonetheless meaningful and has potential value in policymaking.

. . . . End of Solution . . . .

Appendices

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